

EECS 863
Homework

1. A leaky bucket flow control scheme works at the entry point of the network to regulate the offered traffic. Packets arriving to the leaky bucket must wait in a queue for a permit before entering the network. Permits are generated independently from the offered traffic. Thus, upon arrival a packet will be sent if a permit is available otherwise it waits in the buffer for the arrival of a permit. Permits arriving to a system with no packets ready for transmission will be saved in a queue. The permit queue is limited to W permits. A permit is discarded if it arrives when the permit queue is full. For this problem assume that packets arrive to the system according to a Poisson process at a rate λ . Also assume that the permits arrive according to a Poisson process at a rate μ . Let $W=4$ for this problem.

a) Find the steady-state probability mass function for the number of packets waiting to be transmitted.

b) Find the steady-state probability mass function for the number of permits.

[Hint: use a M/M/1 formulation with a careful definition of the states.]

2. A Markovian queueing system with discouraged arrival can be modeled with the following state dependent arrival and departure rates:

$$\mu(n) = \mu$$
$$\lambda(n) = \frac{\lambda}{n+1}$$

a) Draw the state transition diagram.

b) Write the state probabilities, $p(n)$ as a function of μ , λ , and $p(0)$.

c) Find a closed form solution for $p(0)$.

3. Consider a Continuous Time Markov chain with state space $S = \{0, 1, 2\}$ with $\lambda_0 = \lambda_1 = 1$ and $\mu_1 = 1, \mu_2 = 2$

a. Find π_j $j = 1, 2, 3$, using the Q matrix

b. Mapping States into number of customers in the system as follows,

State 1=Empty

State 2=One customer in the system

State 3= Two customers in the system

Find the average number of customers in the system.

c. Find the probability the system is full.